

2. ANNUITIES: COMPOUND FINANCIAL OPERATIONS

2.1. ANNUITY

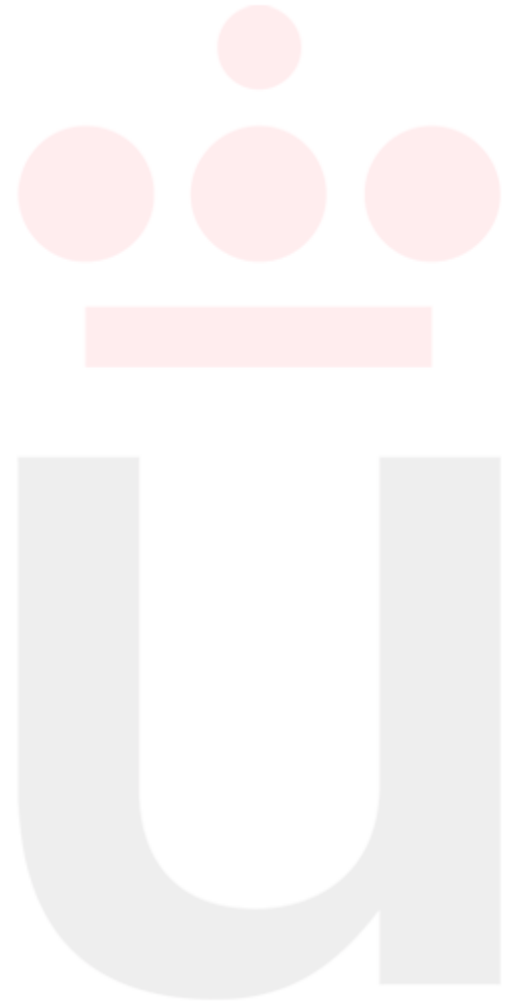
- 2.1.1. Concepts and elements
- 2.1.2. Financial Value of an Annuity

2.2. COMPOUND FINANCIAL OPERATION

- 2.2.1. Definition
- 2.2.2. Financial Equivalence Principle

2.3. PERIODIC ANNUITIES

- 2.3.1. Concept and Elements
- 2.3.2. Classification
- 2.3.3. Financial Values of specific annuities

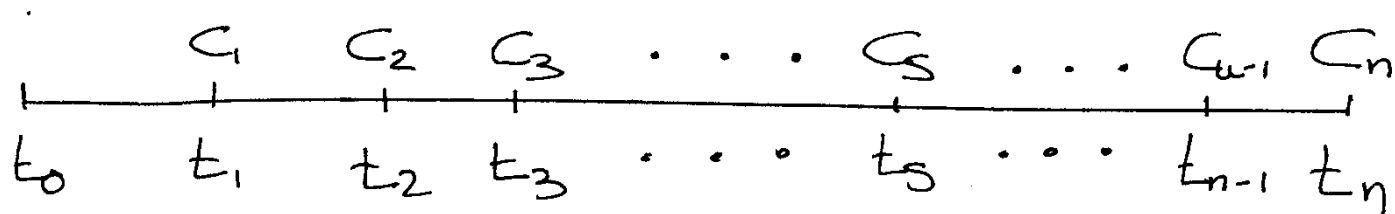


2.1. ANNUITY

2.1.1. CONCEPT AND ELEMENTS

Annuity refers to multiple Financial capitals (cash flows) associated with different intervals or time periods

$$\{(C_1, t_1), (C_2, t_2), \dots, (C_s, t_s), \dots, (C_n, t_n)\}$$



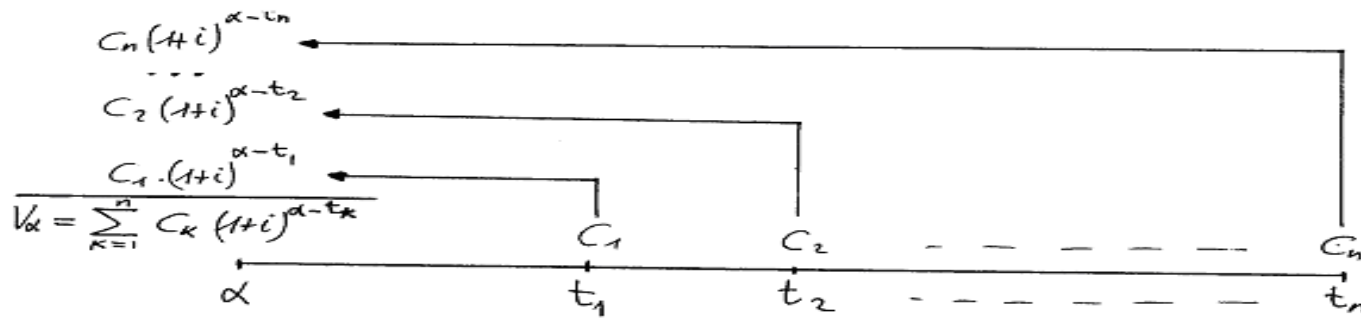
C_i : Terms of the annuity. They are payments (PMTS) or charges made in the financial Operation.

n : number of PMTS

2.1. ANNUITY

2.1.2. Financial Value of an annuity

THE FINANCIAL VALUE OF AN ANNUITY IS the sum of all cash flows or PMTS , valued at the same point in time (Compounded or discounted) based on a particular Financial Law.

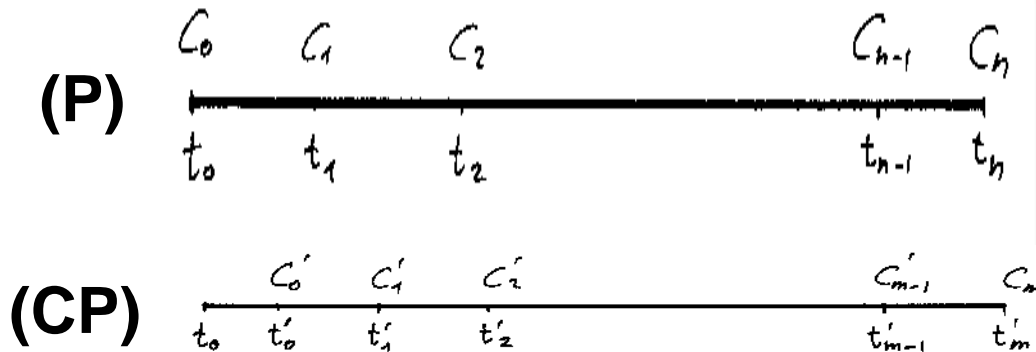


$$V_\alpha = \sum_{k=1}^n C_k (1+i)^{\alpha-t_k}$$

2.2. COMPOUND FINANCIAL OPERATION

2.2.1. Definition

A compound financial Operation is a financial operation in which more than two financial capitals are exchanged (not simultaneously)
For example: a general case is when two annuities are exchanged.



2.2. COMPOUND FINANCIAL OPERATION

2.2.2. FINANCE EQUIVALENCE PRINCIPLE

In every financial operation annuities on both sides must be financially equivalent based on a financial Law. That 's to say that the financial values of both parts must be financially equivalent in accordance with the aforementioned Law

$$\begin{array}{ccc} \underbrace{\{(C_1, t_1), (C_2, t_2), \dots, (C_n, t_n)\}}_{=P} & \underset{f}{\sim} & CP = \underbrace{\{(C'_1, t'_1), (C'_2, t'_2), \dots, (C'_m, t'_m)\}} \\ \downarrow f & \uparrow & \downarrow f \\ & (V_\alpha, \alpha) \underset{f}{\sim} (V'_\beta, \beta) & \end{array}$$

In every financial operation the financial value of one part must coincide with the financial value of the other part in the same point, based on a financial Law.

$$(V_\alpha, \alpha) \underset{f}{\sim} (V'_\alpha, \alpha) \Rightarrow V_\alpha = V'_\alpha \quad \forall \alpha$$

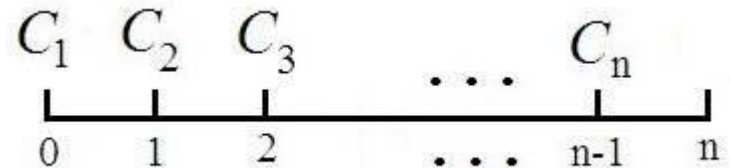
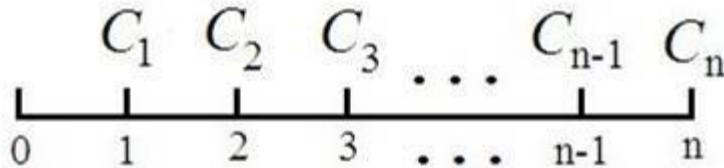
2.3. FINANCIAL PERIODIC ANNUITIES

2.1.1. The concept:

We consider a financial annuity to be periodic when the time period that exists between its terms or cash-flows is always the same.

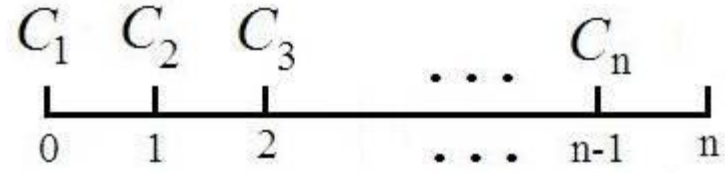
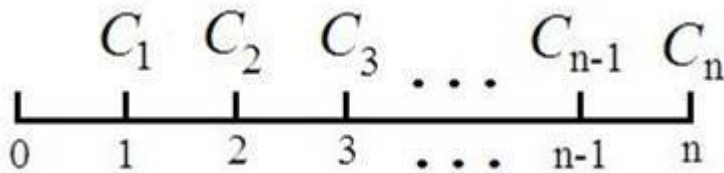
Therefore, every cash-flow is associated with an interval which lasts the same amount of time.

EXAMPLES OF PERIODIC ANNUITIES



NOTE: The maturity of the annuity's cash flows can be any period of time. For instance at the beginning, middle or the end of the interval.

2.2.2. ELEMENTS OF A PERIODIC ANNUITY



- **ORIGIN:** inferior end of the interval associated with the expiration of the first cash flow.
- **END:** superior end of the interval associated with the expiration of the last cash flow.
- **DURATION:** time period elapsed between the beginning and the end of the annuity.
- **C:** Annuity's terms (cash flows) .
- **n:** number of cash flows of an annuity.
- **i:** type of interest rate. (Always Compound Interest Law)

2.2.3. CLASSIFICATION OF PERIODIC ANNUITIES

2.2.3. CLASSIFICATION

2.2.3.1. According to the maturity of the cash flows

2.2.3.2. According to the quantity of the cash flows

2.2.3.3. According to the duration of the annuity

2.2.3.4. According to the moment of valuation

2.2.3.1. CLASSIFICATION OF PERIODIC ANNUITIES

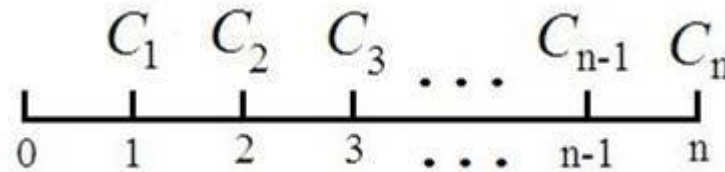
1 Classification: According to the maturity of the of cash flows

ORDINARY ANNUITY

The maturity of the quantities is at the end of the period

The beginning of the annuity: the period before the maturity of the first PMT

The end of annuity : coincides with the maturity of the last PMT

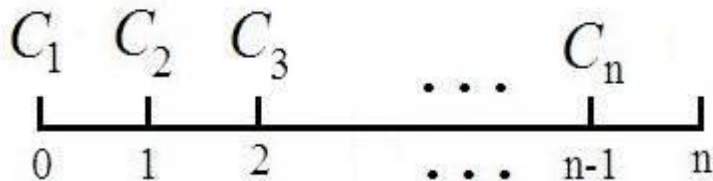


ANNUITY DUE:

The maturity of the cash flows is at the beginning of every period

The beginning of annuity: coincides with the maturity of the first PMT

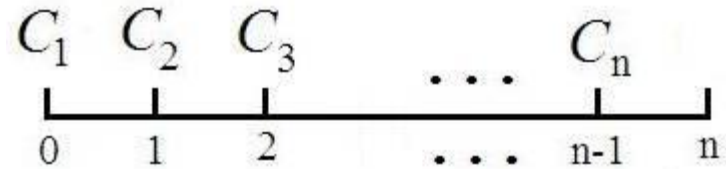
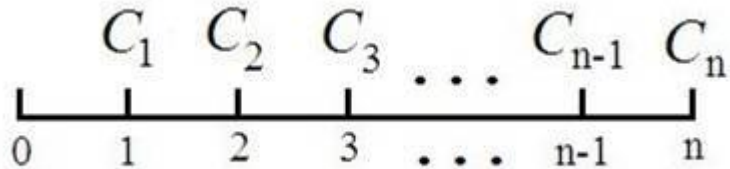
The end of an annuity is (located)one period after the maturity of the last PMT



2.2.3.2. CLASSIFICATION OF PERIODIC ANNUITIES

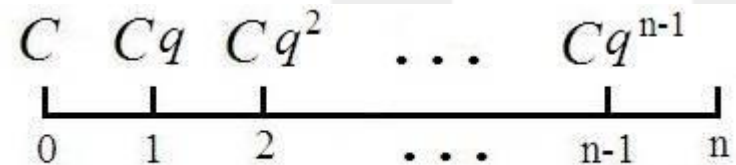
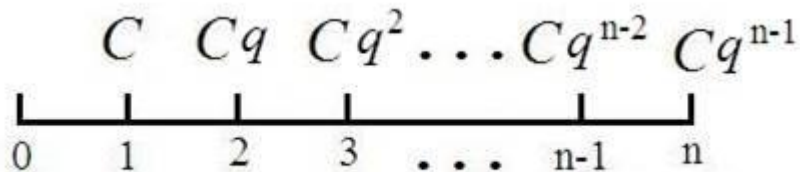
2 Classification: According to the Quantity of the PMTS

CONSTANT ANNUITIES: Are those in which the quantity of all cash –flows is the same



GEOMETRIC ANNUITIES (Variable Cash-Flows)

Are those in which the C-FS vary geometrically (the PMTS increase or decrease at a percentage related to the previous percentage)

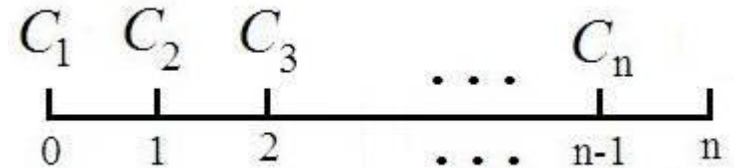
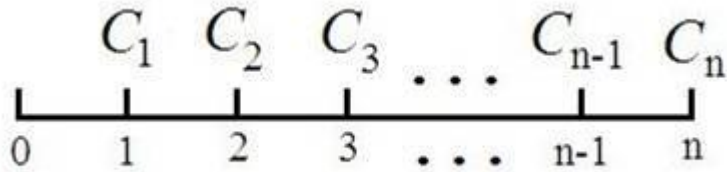


2.2.3.3. CLASSIFICATION OF PERIODIC ANNUITIES

3 Classification: According to the Duration of the Annuity

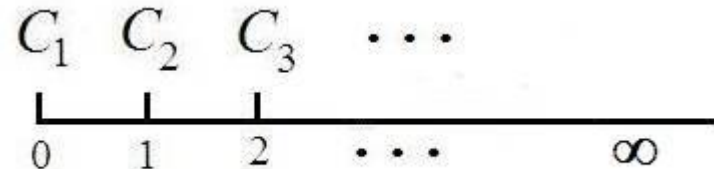
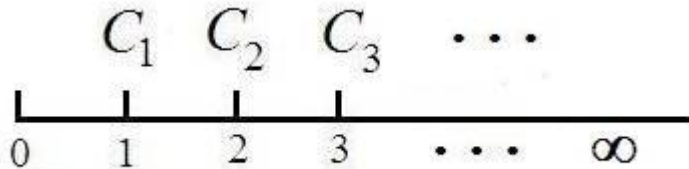
TEMPORARY ANNUITIES.

The number of the terms is finite



PERPETUAL ANNUITIES.

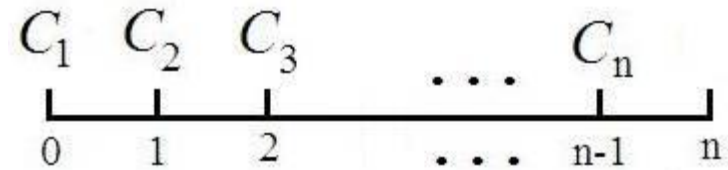
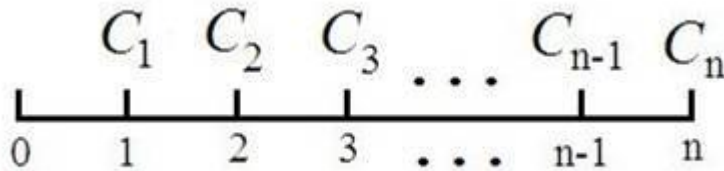
The number of the terms tends toward infinity



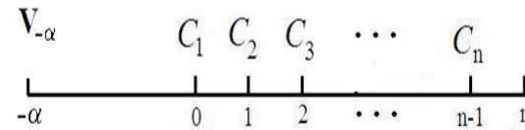
2.2.3.4. CLASSIFICATION OF PERIODIC ANNUITIES

4 Classification: According to the moment of annuity valuation

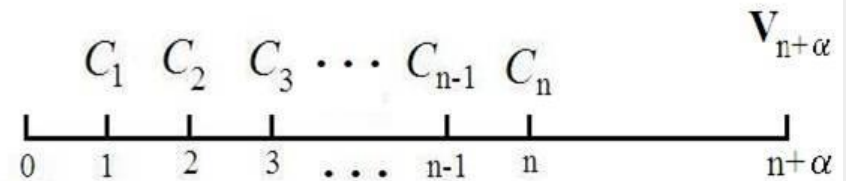
The point of valuation is found between the origin and the end of annuity



The point of valuation is previous to the origin of the annuity



The point of valuation is after the end of the annuity



FINANCIAL VALUES OF SOME PARTICULAR ANNUITIES

Some Ideas

- The Financial Law applied to Annuities is Compound Interest Law at a Constant Interest Rate.
- PRESENT VALUE OR DISCOUNTED VALUE (valuation of the annuity at origin)

We are calculating the financial value of the annuity at the beginning. $t = 0$

Future Value (Valuation of the annuity at the end)

We are calculating the financial value of the annuity at the end. $t=n$

$$V_n = V_o(1+i)^n$$

$$V_o = V_n(1+i)^{-n}$$

FINANCIAL VALUES OF SOME PARTICULAR ANNUITIES

1^o Classification: According to the maturity of the PMTS

ORDINARY ANNUITY

ANNUITY DUE

2^o Classification: According to the quantity of the PMTS

CONSTANT

GEOMETRIC

ANNUITIES

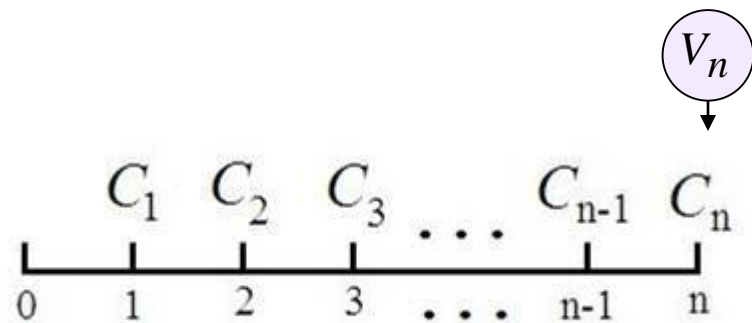
3^o Classification: According to the duration of the annuity

TEMPORARY

PERPETUAL

4^o Classification: According to the moment of valuation

OVERVIEW OF AN ORDINARY ANNUITY- FV



$$V_n = C(1+i)^{n-1} + C(1+i)^{n-2} + C(1+i)^{n-3} + \dots + C$$

$$V_n = C \underbrace{\left((1+i)^{n-1} + (1+i)^{n-2} + (1+i)^{n-3} + \dots + 1 \right)}_{r=(1+i)^{-1}}$$

$$V_n = C \frac{(1+i)^{n-1} - (1+i)^{-1}}{1 - (1+i)^{-1}} = C \frac{(1+i)^{-1} ((1+i)^n - 1)}{(1 - (1+i)^{-1})}$$

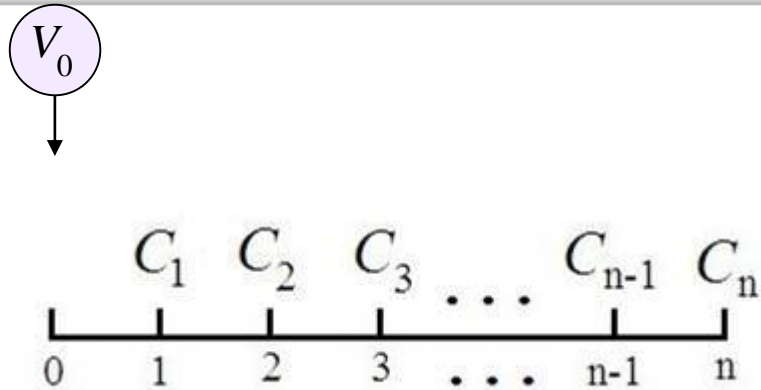
$$V_n = C \frac{(1+i)^n - 1}{(1+i)(1 - (1+i)^{-1})} = C \frac{(1+i)^n - 1}{(1+i) - 1} = C \frac{(1+i)^n - 1}{i}$$

FUTURE VALUE

$$V_n = C \frac{(1+i)^n - 1}{i} = C \cdot S_{\overline{n}|i}$$

$$S = \frac{a_1 - a_n \cdot r}{1 - r}$$

OVERVIEW OF AN ORDINARY ANNUITY- PV



PRESENT VALUE

$$V_0 = C \frac{1 - (1+i)^{-n}}{i} = C \cdot a_{\overline{n}|i}$$

$$V_0 = C(1+i)^{-1} + C(1+i)^{-2} + C(1+i)^{-3} + \dots + C(1+i)^{-n}$$

$$V_0 = V_n(1+i)^{-n}$$

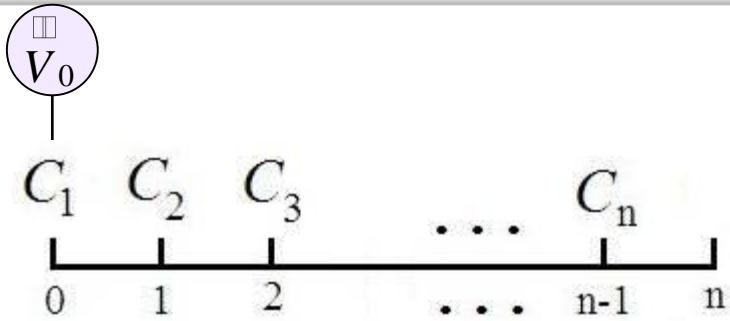
$$V_0 = C \frac{(1+i)^n - 1}{i} (1+i)^{-n}$$

$$V_0 = C \frac{1 - (1+i)^{-n}}{i}$$

Property

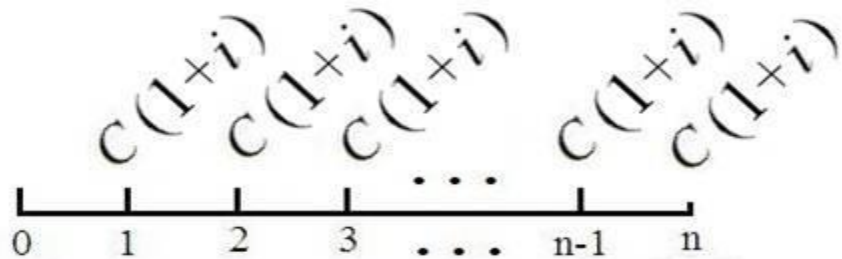
$$S_{\overline{n}|i} = a_{\overline{n}|i} (1+i)^n$$

OVERVIEW OF AN ANNUITY DUE - PV



$$\ddot{V}_0 = C + C(1+i)^{-1} + C(1+i)^{-2} + \dots + C(1+i)^{-n}$$

Equivalent Ordinary Annuity



$$\ddot{V}_0 = (1+i)V_0 \qquad \ddot{V}_0 = C(1+i) \frac{1-(1+i)^{-n}}{i}$$

PRESENT VALUE

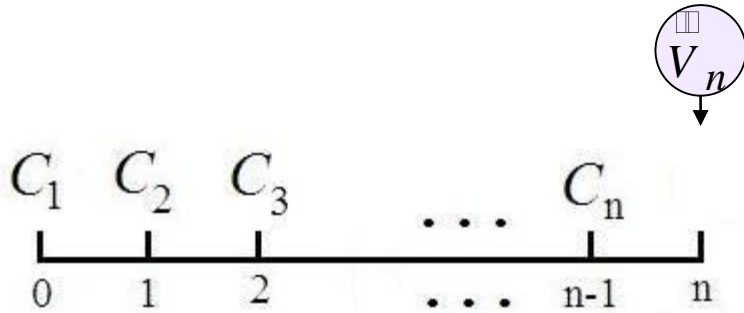
$$\ddot{V}_0 = C(1+i) \frac{1-(1+i)^{-n}}{i} = C \cdot \ddot{a}_{n|i}$$

Properties

$$\ddot{S}_{n|i} = \ddot{a}_{n|i} (1+i)^n$$

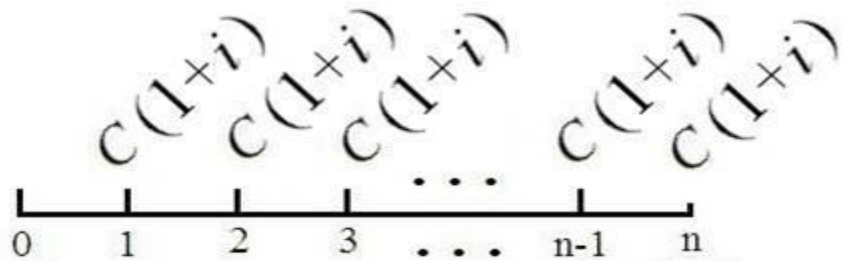
$$\ddot{a}_{n|i} = a_{n|i} (1+i)$$

OVERVIEW OF AN ANNUITY DUE - FV



$$\ddot{V}_n = C(1+i)^n + C(1+i)^{n-1} + C(1+i)^{n-2} + \dots + C(1+i)$$

Equivalent ordinary annuity



$$\ddot{V}_n = C(1+i) \frac{(1+i)^n - 1}{i}$$

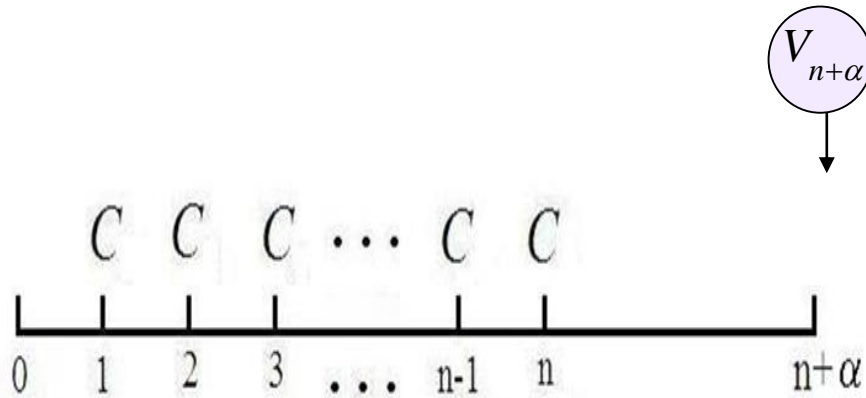
FUTURE VALUE

$$\ddot{V}_n = C(1+i) \frac{(1+i)^n - 1}{i} = C \cdot \ddot{S}_{n|i}$$

Property

$$\ddot{S}_{n|i} = S_{n|i} (1+i)$$

FINANCIAL VALUE AFTER THE END



Financial Value after the end

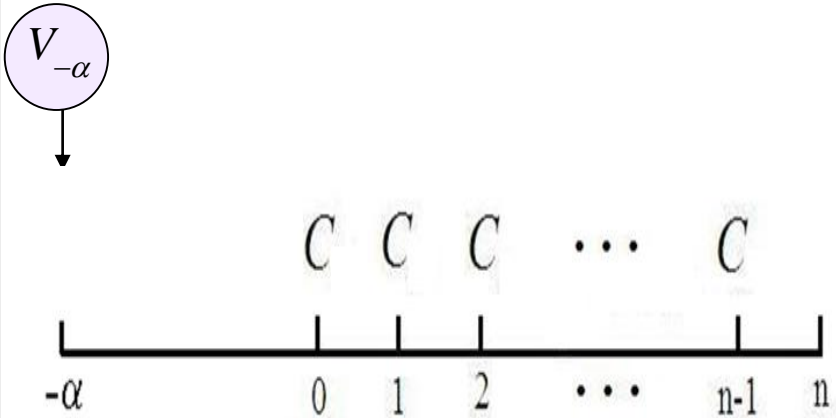
$$V_{n+\alpha} = C \frac{(1+i)^n - 1}{i} \cdot (1+i)^\alpha$$

$$V_{n+\alpha} = V_n \cdot (1+i)^\alpha$$

$$V_{n+\alpha} = C \cdot S_{\overline{n}|i} \cdot (1+i)^\alpha$$

$$V_{n+\alpha} = C \frac{(1+i)^n - 1}{i} \cdot (1+i)^\alpha$$

FINANCIAL VALUE BEFORE THE BEGINNING



Financial Value before the beginning

$$V_{-\alpha} = C(1+i) \frac{1-(1+i)^{-n}}{i} (1+i)^{-\alpha}$$

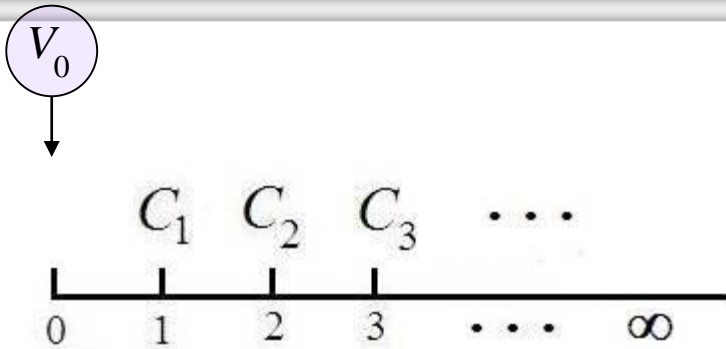
$$V_{-\alpha} = \ddot{V}_0 \cdot (1+i)^{-\alpha}$$

$$V_{-\alpha} = C \cdot \ddot{a}_{n|i} \cdot (1+i)^{-\alpha}$$

$$V_{-\alpha} = C(1+i) \frac{1-(1+i)^{-n}}{i} (1+i)^{-\alpha}$$



OVERVIEW OF A PERPETUAL ORDINARY ANNUITY



$$V_0 = C(1+i)^{-1} + C(1+i)^{-2} + C(1+i)^{-3} + \dots$$

$$V_0 = \lim_{n \rightarrow \infty} C \cdot a_{\overline{n}|i}$$

$$V_0 = \lim_{n \rightarrow \infty} C \frac{1 - (1+i)^{-n}}{i}$$

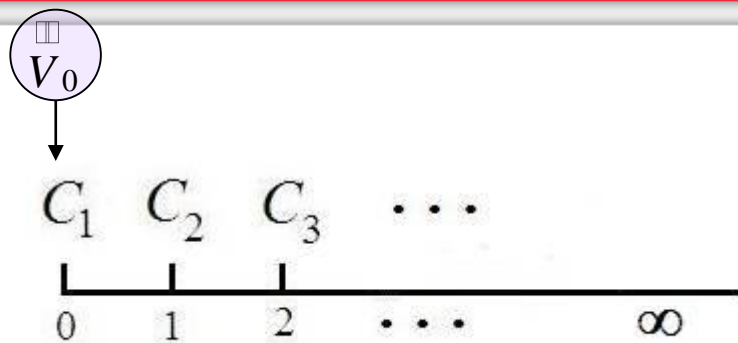
$$V_0 = C \frac{1 - (1+i)^{-\infty}}{i}$$

$$V_0 = C \frac{1}{i}$$

PRESENT VALUE

$$V_0 = C \frac{1}{i} = C \cdot a_{\overline{\infty}|i}$$

OVERVIEW OF A PERPETUAL ANNUITY DUE



$$\ddot{V}_0 = C + C(1+i)^{-1} + C(1+i)^{-2} + C(1+i)^{-3} + \dots$$

$$\ddot{V}_0 = \lim_{n \rightarrow \infty} C \cdot \ddot{a}_{n|i}$$

$$\ddot{V}_0 = \lim_{n \rightarrow \infty} C(1+i) \frac{1-(1+i)^{-n}}{i}$$

$$\ddot{V}_0 = C(1+i) \frac{1-(1+i)^{-\infty}}{i}$$

$$\ddot{V}_0 = C(1+i) \frac{1}{i}$$

PRESENT VALUE

$$\ddot{V}_0 = C(1+i) \frac{1}{i} = C \cdot \ddot{a}_{\infty|i}$$

Property

$$\ddot{a}_{\infty|i} = (1+i)a_{\infty|i}$$